Diffraction Method to Test the Weak Equivalence Principle for Neutron

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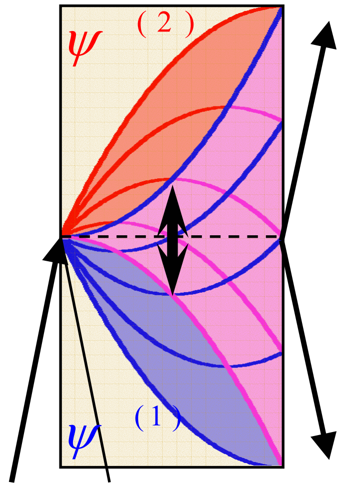
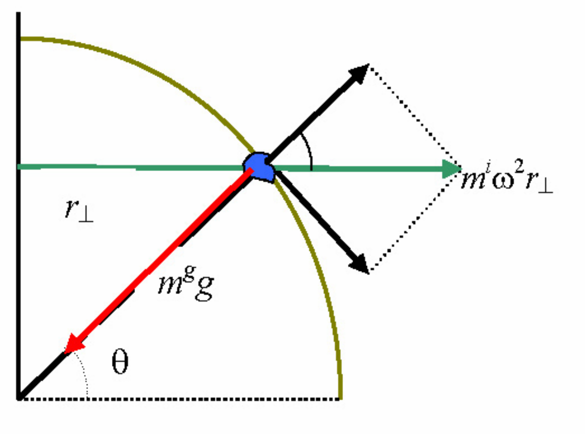


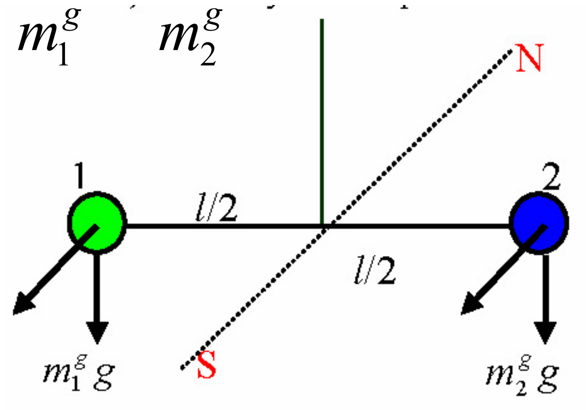
Main idea (Eõtvõs experiment)

Centrifugal force $\mathbf{F_r} \sim m^i$
Gravitational force $\mathbf{F_G} \sim m^g$



Torsion balance will be in equilibrium if

$$\frac{m_1^i}{m_1^g} = \frac{m_2^i}{m_2^g}$$



$$\frac{m_1^i}{m_1^g} - \frac{m_2^i}{m_2^g} \le 5\cdot 10^{-9}$$

R. v. Eõtvõs utilized Earth rotation (1908)

$$\frac{m_1^i}{m_1^g} - \frac{m_2^i}{m_2^g} \le 10^{-12}$$

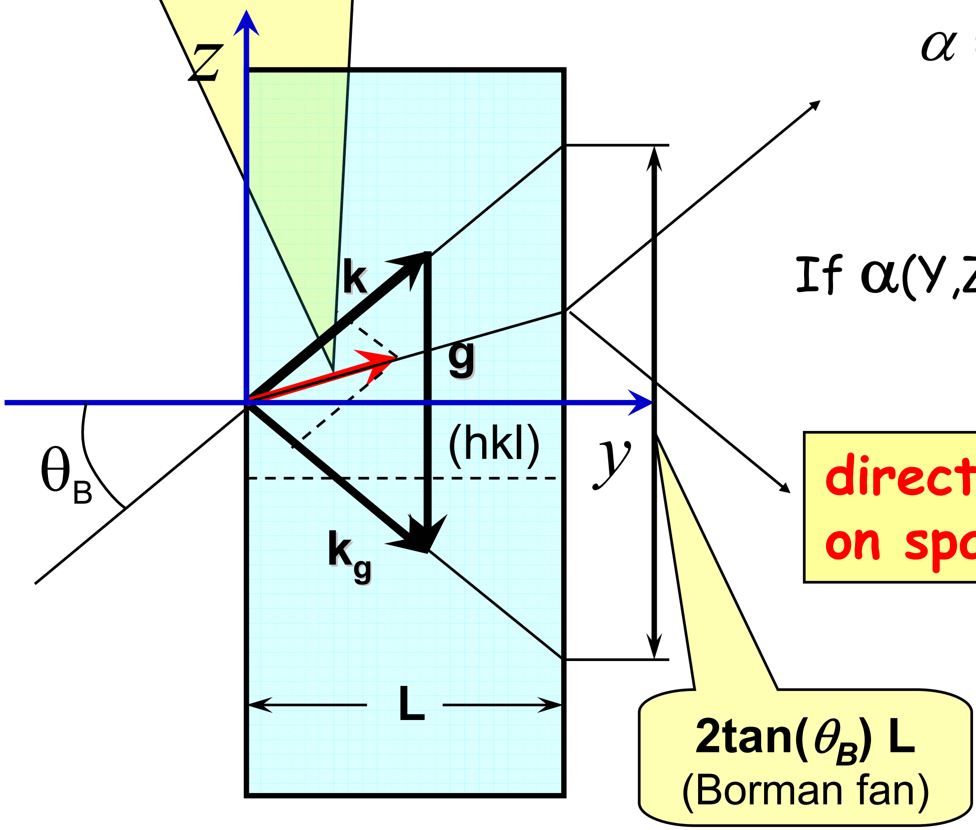
V.B. Braginsky and V.I. Panov, Sov JETPh, (1971) **61** 873 utilized Earth rotation about the Sun

$$\frac{m^g - m^i}{m^i} \le 2\cdot 10^{-4}$$

Best result for elementary particle (neutron), J. Schmiedmayer NIM A **284** 59 (1989)

Neutron trajectory for Laue diffraction

$\mathbf{j} = \hbar/m(|a_g(\alpha)|^2\mathbf{k_g} + |a_0(\alpha)|^2\mathbf{k})$



Amplitudes a_g and a_0 depend on a deviation from Bragg condition

$$\alpha = \frac{2(\Delta\mathbf{k}_0 \cdot \mathbf{g})}{k_0^2}, \quad \Delta\mathbf{k}_0 = \mathbf{k}_0 - \mathbf{g}/2$$

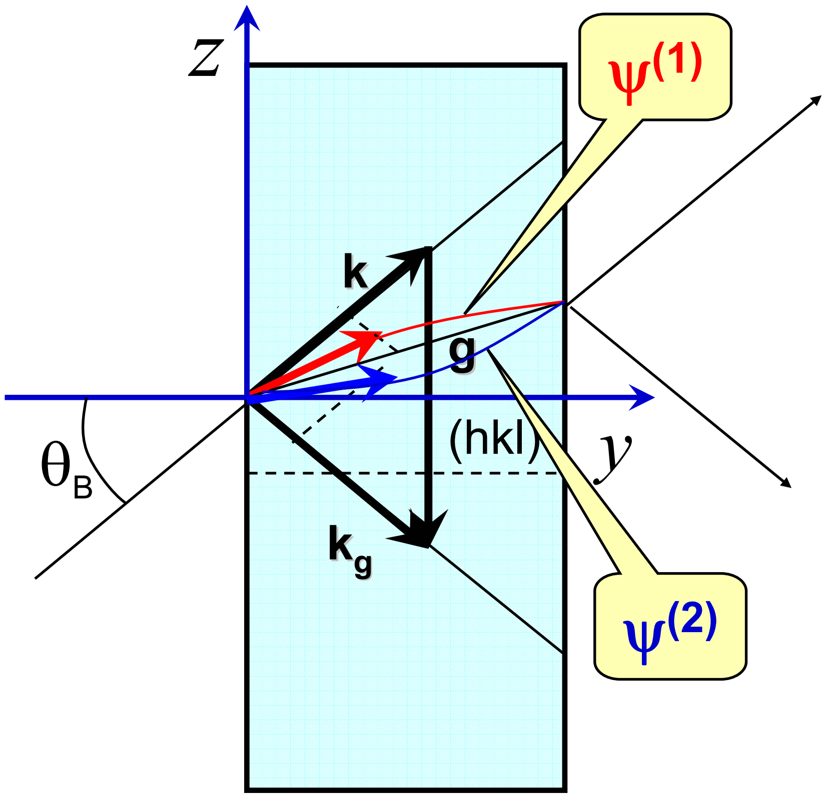
$a_g(\alpha)$ and $a_0(\alpha)$

If $\alpha(Y,Z)$ → $a_g(Y,Z)$ and $a_0(Y,Z)$

direction of neutron current depends on spatial coordinates. j(Y,Z)

Neutron in deformed crystal

N.Kato , J. Phys. Soc. Japan (1963) **19**, 971



"Kato force", determined by the crystal deformation

$$f_k(y,z) = -\frac{k_0}{4\cos\theta_B}\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial y}\right)\alpha(y,z),$$

For small deformation the neutron trajectories is determined by

$$\frac{\partial^2 z}{\partial y^2} = \pm\frac{\tan(\theta_B)}{m_0} f_k(y,z)$$

$\psi^{(1)}$ or $\psi^{(2)}$

$$m_0 = \frac{2F_g d}{V}$$

External force F_n

$$\Delta\mathbf{k}_0 = \mathbf{k}_0 - \mathbf{g}/2$$

Deformation change the reciprocal lattice vector ***g***

$$\Delta\mathbf{k}_0(y,z) = \mathbf{k}_0 - \mathbf{g}(y,z)/2$$

External force change neutron energy

$$\Delta\mathbf{k}_0(y,z) = \mathbf{k}_0(y,z) - \mathbf{g}/2$$

External force $F_n || g$ equivalent to gradient of interplanar distance

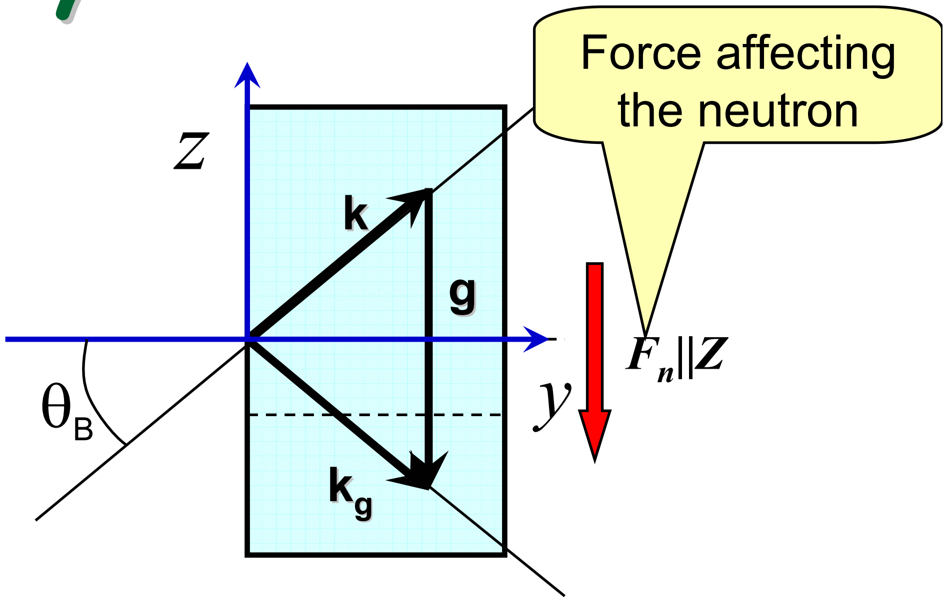
$$d = d_0(1+\xi_F \cdot z)$$

$$\xi_F = \frac{F_n}{2E_n}$$

$$\boxed{f_k = \tan(\theta_B)\frac{\pi}{d}\frac{F_n}{2E_n}}$$

Neutron trajectory

Laue diffraction	Free neutron
$\frac{\partial^2 z}{\partial y^2} = \boxed{\pm \frac{\tan^2(\theta_B)}{m_0} \frac{\pi}{d}} \frac{F_n}{2E_n}$	$\frac{\partial^2 z}{\partial y^2} = \frac{F_n}{2E_n}$



Gain factor $\boxed{K_d = \pm \frac{\tan^2(\theta_B)}{m_0} \frac{\pi}{d}}$

$\theta_B \sim (84-87)^0$

quartz (110) $\rightarrow \tan^2(\theta_B) \times 1.8 \cdot 10^5 \rightarrow (10^7 - 10^8)$

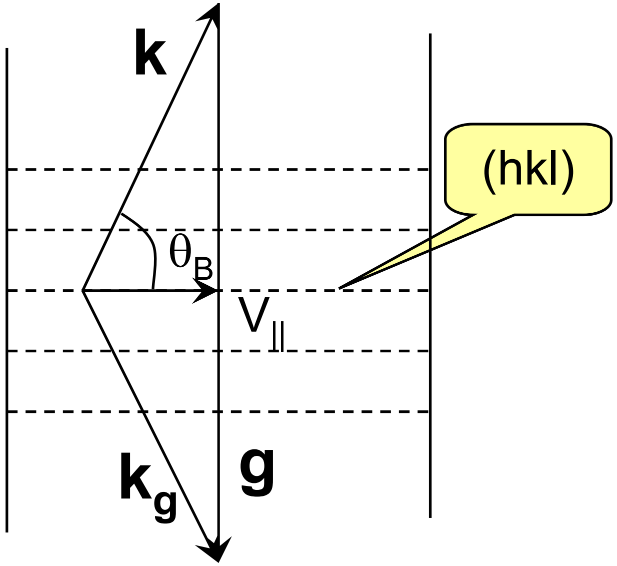
(200) $\rightarrow \tan^2(\theta_B) \times 1.4 \cdot 10^6 \rightarrow (10^8 - 10^9)$

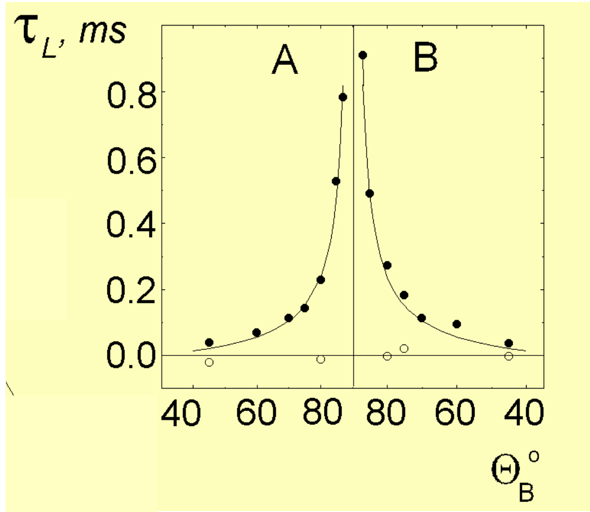
10 cm crystal ⇔ **1 km free flight**

Effect of diffracted neutron "slowing down" in a crystal

Time of neutron flight is determined by

$\boxed{\tau = L/v_{||}}$





$$|\mathbf{v}_{||}| = \frac{\hbar k \cos\theta_B}{m} = v\cos\theta_B \approx v\left(\frac{\pi}{2} - \theta_B\right) \quad \text{for} \quad \theta_{\mathrm{B}} \sim \frac{\pi}{2}$$

For instance $\theta_B = 87^0$ ⟹ $V_{||} = V(\pi/2 \quad \theta_B) \approx V/20$

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Effective Mass of Neutrons Diffracting in Crystals

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(Received 25 November 1985)

Neutrons propagating in a crystal under diffraction conditions exhibit an effective inertial mass which is lower by 5–6 orders of magnitude than their vacuum rest mass and of both positive and negative sign. This is verified experimentally by measurement of the enormously enhanced deflection of neutrons subjected to a magnetic force while passing through a silicon crystal.

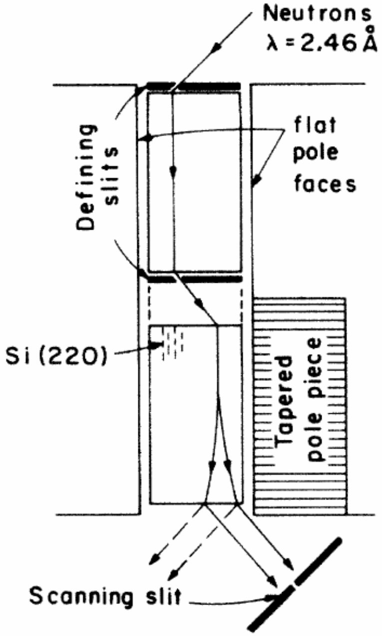


FIG. 1. The experimental arrangement used in the study of the reduced inertial mass of neutrons in crystals.

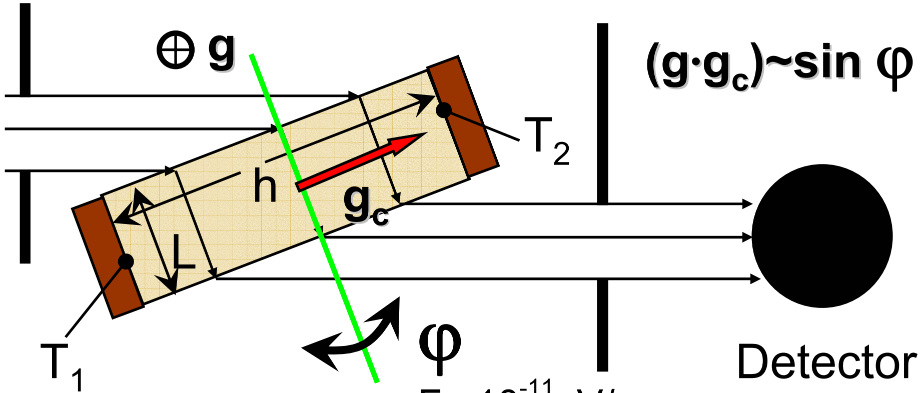
Effective mass of neutron

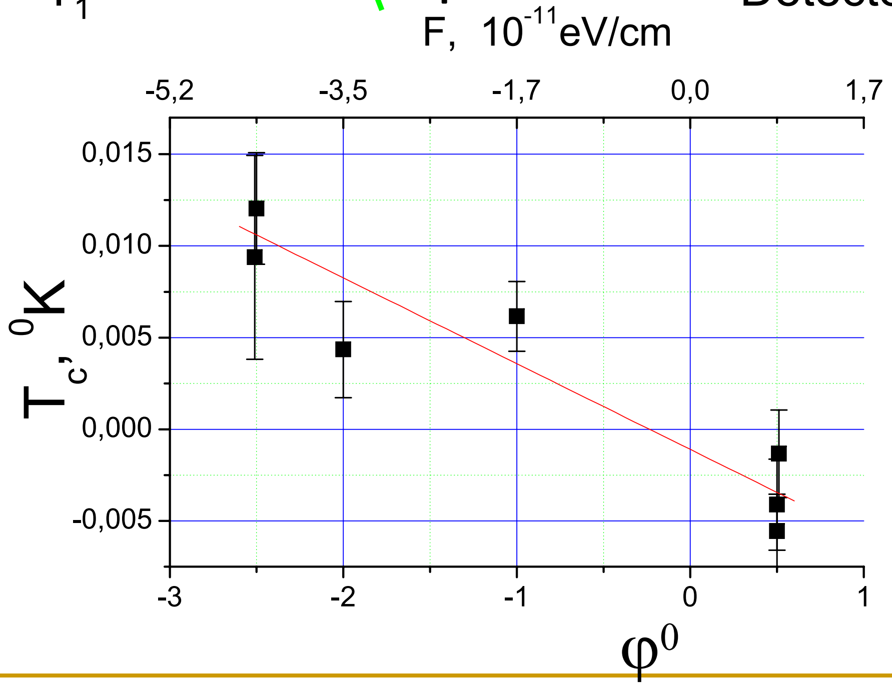
$$m/m^* = \pm\frac{E_g}{2V_g} \equiv \pm\frac{1}{m_0}\frac{\pi}{d}$$

$$K_d = \pm\frac{\tan^2(\theta_B)}{m_0}\frac{\pi}{d}$$

$$\left.\begin{array}{l} m/m^* \approx 10^5 \\ \tan^2(\theta_B) \approx 10^3 \end{array}\right\} \quad \boxed{\approx 10^8}$$

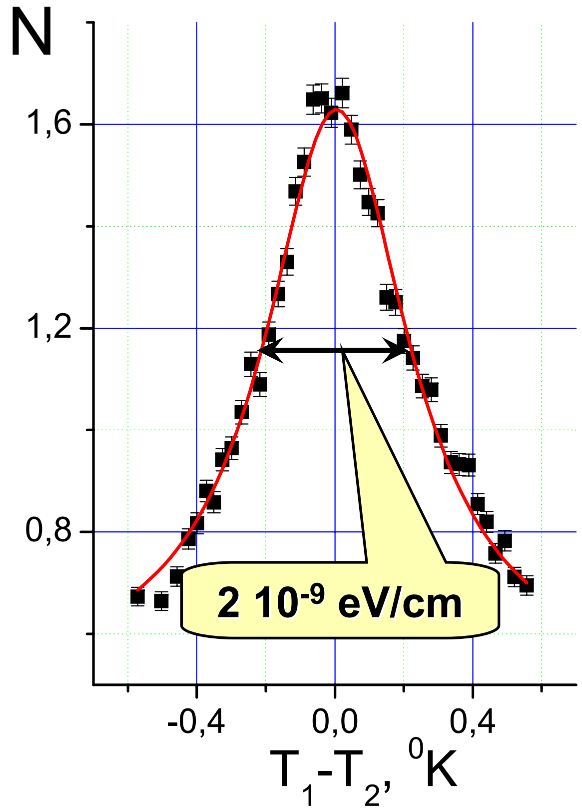
Demonstration experiment



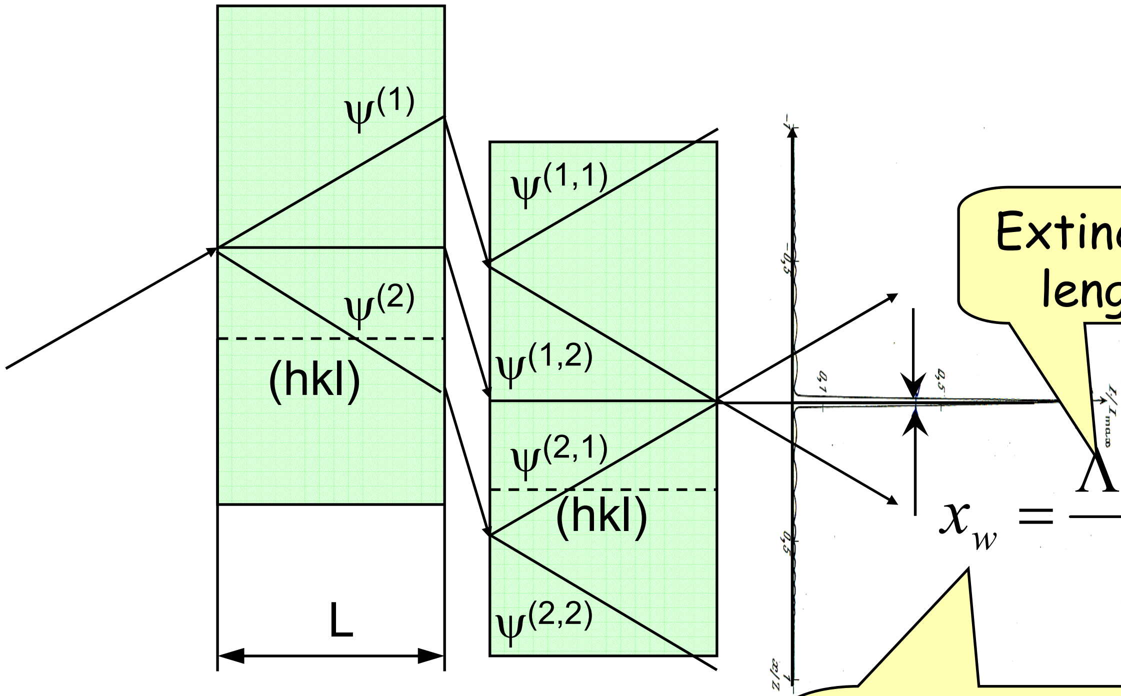


(110) quartz plane

L=14 cm, $\theta_B = 76^0$, $\tan(\theta_B) = 4$



Diffraction focusing of neutron beam



Quartz

(110) ➡ $x_w = 7\,\mu m$

(200) ➡ $x_w = 50\,\mu m$

Extinction length

$$x_w = \frac{\Lambda \tan(\theta_B)}{2\pi} = \frac{1}{2m_0}$$

Doesn't depend on Bragg angle and crystal length

V.L.Indenbom, I.Sh.Slobobedsky, K.G.Truni, Sov. JETP, 66(3), 1110 (1974)

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Dynamical neutron diffraction in a thick-crystal interferometer

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(Received 7 June 1985)

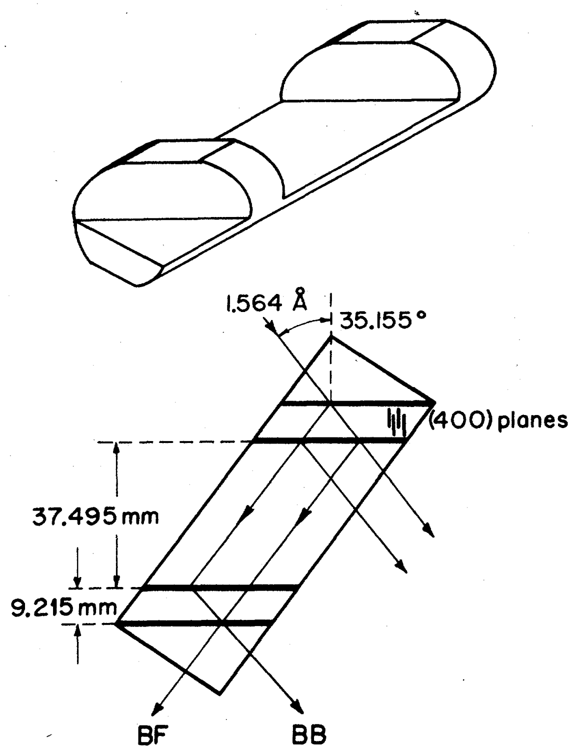
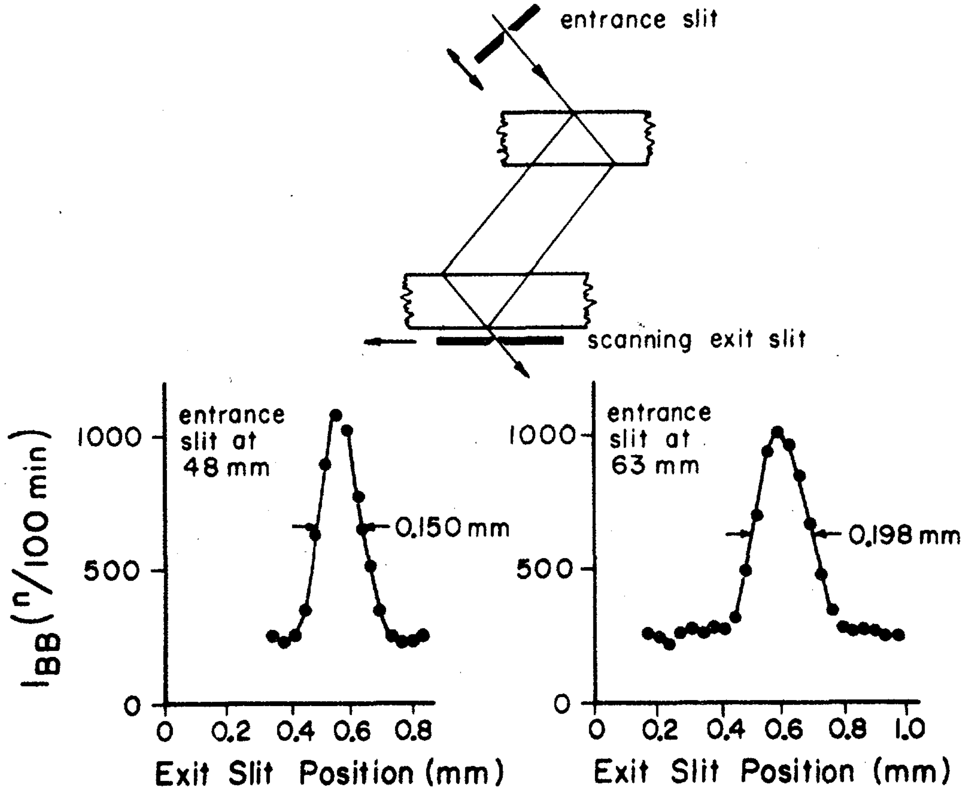


FIG. 1. Illustration of the MIT two-crystal interfe
cut from a single perfect crystal of silicon.



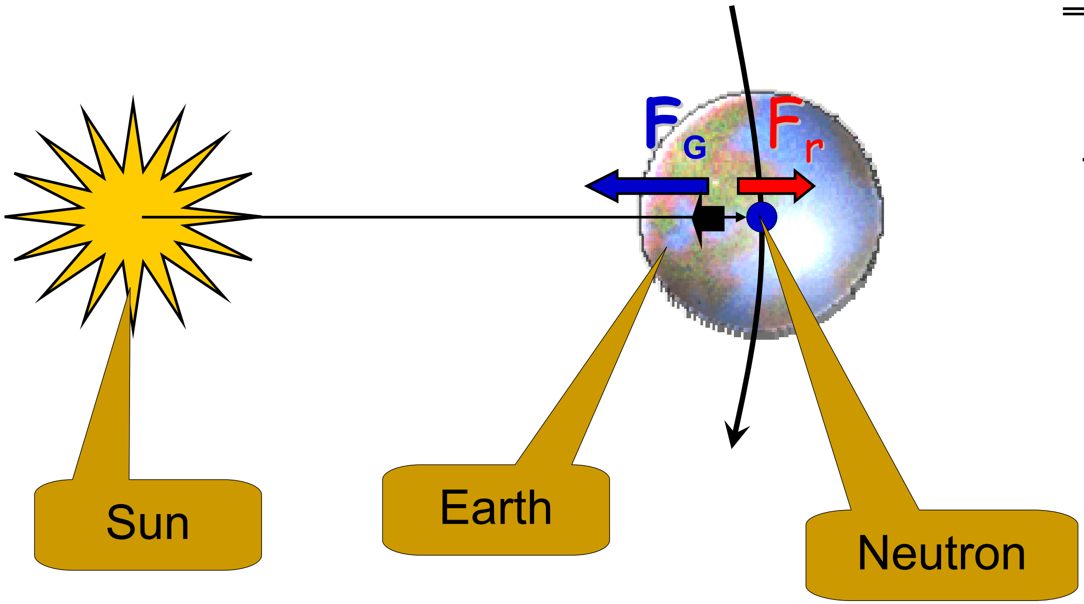
Spot size can be ~0.1mm and coincides with the theory

Estimation of sensitivity

Let crystals size 10x10x5 cm³, θ_B=84⁰ (tan(θ_B) =10),
Δ_W=0.1mm, $\Phi=10^9$ n/(s·cm²·Å)

	F_W , eV/cm	N, n/s	***δF, eV/cm per day***
Quartz (110) λ=4.9Å	$1.5 \cdot 10^{-13}$	2200	**$5 \cdot 10^{-18}$**
Quartz (200) λ=4.2Å	$0.3 \cdot 10^{-13}$	250	**$3 \cdot 10^{-18}$**
Silicon (220) λ=3.8Å	$2 \cdot 10^{-13}$	600	**$12 \cdot 10^{-18}$**

m_i/m_G experiment for neutron



$F_G = F_r$ for the Earth

If the ratio of inertial
and gravitational masses
for neutron and Earth are different

$$\frac{m_n^i}{m_n^g} \neq \frac{m_\otimes^i}{m_\otimes^g}$$

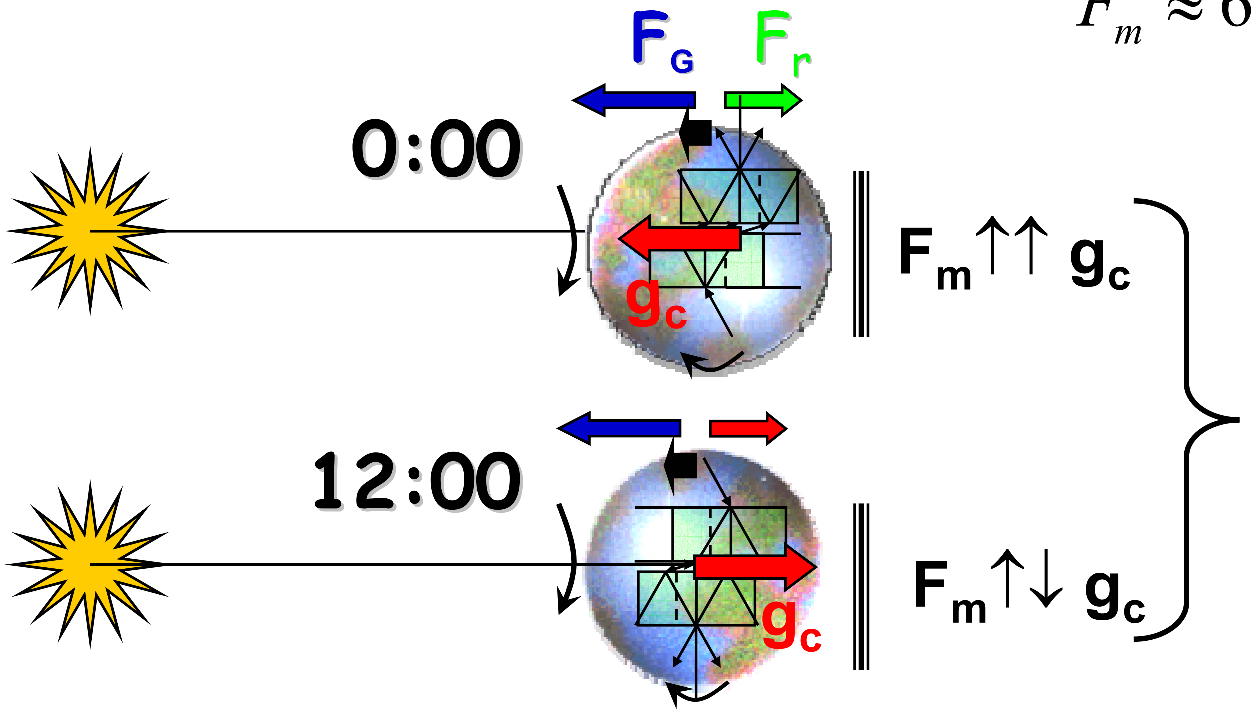
$F_G \neq F_r$ for the neutron

$$F_m \equiv F_G - F_i = G \cdot \frac{m_\odot m_n^g}{R^2}\left(1 - \frac{m_n^i / m_n^g}{m_\otimes^i / m_\otimes^g}\right)\Bigg|_{m_\otimes^i / m_\otimes^g \equiv 1} \approx 6 \cdot 10^{-13}\left(1 - \frac{m_n^i}{m_n^g}\right) eV/cm$$

PNPI

m_i/m_G experiment for neutron

$$F_m \approx 6\cdot 10^{-13}\left(1-\frac{m_n^i}{m_n^g}\right) eV/cm$$



Force change the sign in the laboratory coordinate system

We should see daily oscillations

The setup sensitivity can be
$\delta F \sim 5\ 10^{-18}$ ***eV/cm per day***

$\delta(m_n^i/m_n^g) \sim 10^{-5}$ per day

Present accuracy $\delta(m_i/m_G) \sim 1.7\ 10^{-4}$

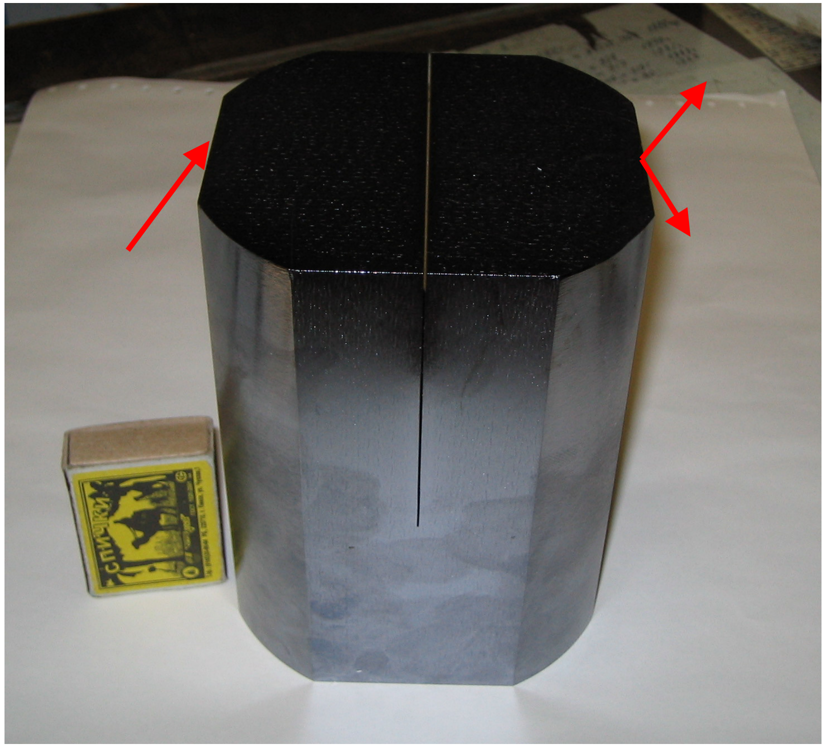
J. Schmiedmayer (1989)

Current status

- Setup project is ready
- Simulation is in progress.
- Part of equipment is purchased
  - Inclinometer (sensitivity 0.001")
  - Water thermostat (stability 0.01 K)
- Silicon crystals is ready. $\omega_m \cong 1"$
  Working plane – (220)

  Single crystal size 47x90x80 mm^3,

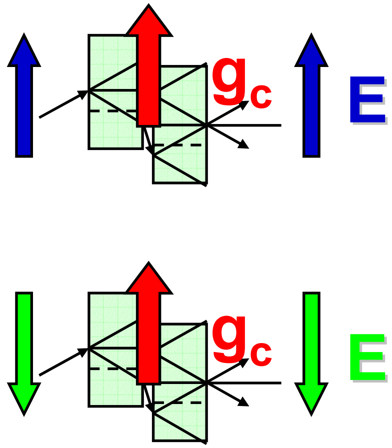
  $\delta(m_n^i/m_n^g) \sim 3 \cdot 10^{-5}$ per day



Plans

- 2009 – finishing the setup building, prepare another crystal (70x140x100 mm^3) $\delta(m_n^i/m_n^g) \sim 1.2 \cdot 10^{-5}$ per day
- 2010 – start the measurement

e_n experiment



$F_m = +e_n E$

$F_m = -e_n E$

Let E=30 kV/cm

$\delta(e_n) \sim 2\ 10^{-22}$e per day

current limit*) - $e_n = 10^{-21}$e

*) J.Baumann, R.Gahler, J.Kalus, W.Mampe, PR **D37**, 3107 (1988)

Problem –

Quartz crystal is a piezoelectric – crystal deformation accompanied with electric field

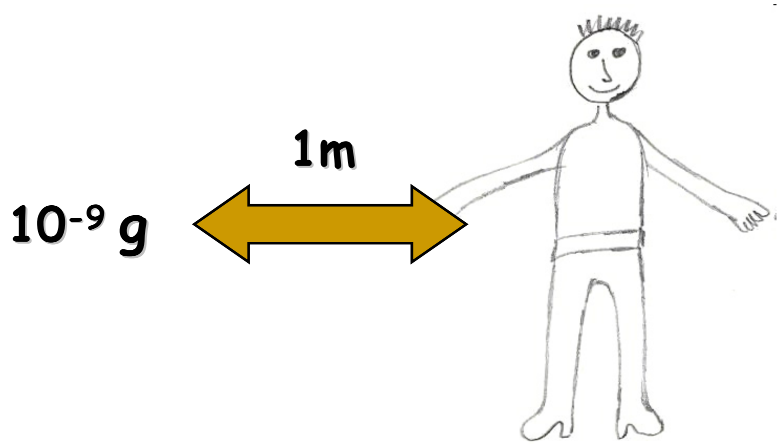
Silicon is semiconductor – no electric field inside the crystal

We should looking for other perfect crystal – sapphire?

Conclusion

- The close to $\pi/2$ Laue diffraction is extremely sensitive instrument for the investigation of fundamental neutron interaction.

Resolution to the external field can be better 10^{-13} eV/cm and sensitivity ~ 10^{-18} eV/cm, (10^{-9} g)



$$\left.\begin{array}{l}\delta(m_i/m_G) \sim 10^{-6} \\ \delta(e_n) \sim 10^{-23}e\end{array}\right\}$$

About 100 times better than current value